Markowitz 2.0
Asset Allocation per il 21° secolo

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Asset Allocation in the 21st Century

Paul D. Kaplan, Ph.D., CFA
Quantitative Research Director, Morningstar Europe, Ltd.
Harry Markowitz and Mean-Variance Optimization

Harry Markowitz, Nobel Prize Winner
Asset Allocation in 1952

“In our analyses the [portfolio weights] might represent individual securities or they might represent aggregates such as, say, bonds, stocks, and real estate.”

Harry Markowitz (1952)
“I think the most important thing that happened between 1959 and the present is the notion of doing your analysis on asset classes in the first instance. This has become part of the infrastructure that we now rely on. I had a rationale, and so on. Now we have an industry.”

Harry Markowitz (2010)
The Asset Allocation Paradigm

Asset Classes

- Equities
- Fixed Income
- Real Estate

Managers/Funds

- Active Equity Fund 1
- Active Equity Fund 2
- Equity Index Fund
Methods for Selecting Asset Class Weights

- Naïve approach ($1/n$)
- Market capitalization (Capital Asset Pricing Model, CAPM)
- Optimization
  - Markowitz 1952
  - Markowitz 2.0 (Kaplan & Savage 2010)
Market Capitalization Weights:
Summer 2010 ~$73.6 Trillion

- US Large Cap Growth: 7.3%
- US Large Cap Value: 7.4%
- US Small Cap Growth: 0.6%
- US Small Cap Value: 0.6%
- US Investment Grade Bonds: 20.8%
- TIPS: 0.8%
- US High Yield: 1.1%
- Non-US High Yield: 0.6%
- Non-US Investment Grade Bonds: 30.2%
- Direct Real Estate: 9.8%
- Private Equity: 2.3%
- Emerging Market Equity: 4.3%
- Non-US Equity: 14.2%
- Non-US Investment Grade Bonds: 30.2%
- Non-US High Yield: 0.6%

Estimates are not guaranteed.
Harry Markowitz’s Mean–Variance Optimization

This procedure is viewed as the gold standard for developing an optimal asset allocation.

Mean-Variance Inputs

Mean-Variance Optimizer

Mean-Variance Efficient Frontier

Expected Return vs. Standard Deviation

Capital Market Assumptions

- Expected Returns
- Standard Deviations (Risk)
- Correlations
The Efficient Frontier

Each point on the Efficient Frontier represents a combination of asset classes that maximizes return per unit of risk.

This is a graphical representation; plot points are not necessarily meaningful.
Principles of Asset Allocation

- Diversify across asset classes
- Implement each asset class with
  - Low cost index funds
  - Good managers/funds
- Rebalance regularly
- Be patient and stay in for the long-run
**Ibbotson® SBBI®**

Stocks, Bonds, Bills, and Inflation 1926–2009

<table>
<thead>
<tr>
<th>Compound annual return</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Small stocks</td>
<td>11.9%</td>
</tr>
<tr>
<td>Large stocks</td>
<td>9.8</td>
</tr>
<tr>
<td>Government bonds</td>
<td>5.4</td>
</tr>
<tr>
<td>Treasury bills</td>
<td>3.7</td>
</tr>
<tr>
<td>Inflation</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Past performance is no guarantee of future results. Hypothetical value of $1 invested at the beginning of 1926. Assumes reinvestment of income and no transaction costs or taxes.
Compound annual return

- Small stocks: 11.9%
- Large stocks: 9.8%
- Government bonds: 5.4%
- Treasury bills: 3.7%
- Inflation: 3.0%
- 60% Equity 40% Bond: 8.8%

Past performance is no guarantee of future results. Hypothetical value of $1 invested at the beginning of 1926. Assumes reinvestment of income and no transaction costs or taxes.
Diversification Did Work in 2008

Starting Wealth Jan 2008: $100

<table>
<thead>
<tr>
<th>Asset Classes</th>
<th>Very Aggressive</th>
<th>Aggressive</th>
<th>Moderate</th>
<th>Conservative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>100</td>
<td>75</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Bonds</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>65</td>
</tr>
<tr>
<td>Cash</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

End Wealth Dec 2008: $63, $73, $84, $94

Asset classes are represented by the following benchmarks: Stocks: S&P 500, Bonds: BarCap Aggregate Bond Index, Cash: Citigroup Treasury 3-month T-Bill.

Returns shown are hypothetical; indices are unmanaged and not available for direct investment. Assumes reinvestment of all capital gains and dividends and does not account for transactions costs or taxes. Past performance is not indicative of future results.
An event that is inconsistent with past data but that happens anyway
The Black Turkey

“An event that is everywhere in the data—it happens all the time—but to which one is willfully blind.”

A Flock of Turkeys

Nominal price return unless otherwise specified.

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Time Period</th>
<th>Peak to Trough Decline</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. stocks (real total return)</td>
<td>1911-1920</td>
<td>51%</td>
</tr>
<tr>
<td>U.S. stocks (DJIA, daily)</td>
<td>1929-1932</td>
<td>89%</td>
</tr>
<tr>
<td>Long U.S. Treasury bond (real total return)</td>
<td>1941-1981</td>
<td>67%</td>
</tr>
<tr>
<td>U.S. stocks</td>
<td>1973-1974</td>
<td>49%</td>
</tr>
<tr>
<td>U.K. stocks (real total return)</td>
<td>1972-1974</td>
<td>74%</td>
</tr>
<tr>
<td>Gold</td>
<td>1980-1985</td>
<td>62%</td>
</tr>
<tr>
<td>Oil</td>
<td>1980-1986</td>
<td>71%</td>
</tr>
<tr>
<td>Japan stocks</td>
<td>1990-2009</td>
<td>82%</td>
</tr>
<tr>
<td>U.S. stocks (S&amp;P)</td>
<td>2000-2002</td>
<td>49%</td>
</tr>
<tr>
<td>U.S. stocks (NASDAQ)</td>
<td>2000-2002</td>
<td>78%</td>
</tr>
<tr>
<td>U.S. stocks (S&amp;P)</td>
<td>2007-2009</td>
<td>57%</td>
</tr>
</tbody>
</table>

The Limitations of Mean-Variance Analysis

- Fat tails in returns not modeled
- Covariation of returns assumed linear, cannot handle optionality
- Single period investment horizon (arithmetic mean)
- Risk measured by volatility
- These limitations largely due to the flaw of averages
  - Standard deviation is an average of squared deviations
  - Correlation in an average of comovements
The Flaw of the Bell Shaped Curve


Source: Paul D. Kaplan, “Déja Vu All Over Again,” in Morningstar Advisor, February/March 2009
Performance data shown represents past performance. Past performance is not indicative and not a guarantee of future results. Indices shown are unmanaged and not available for direct investment. Performance data does not factor in transaction costs or taxes.
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The Flaw of the Bell Shaped Curve


Mean less $3\sigma$ should occur about once every 1000 observations.

In this time period, 10 of the 995 observations exceed -15%.

Source: Paul D. Kaplan, “Déjà Vu All Over Again,” in Morningstar Advisor, February/March 2009

Performance data shown represents past performance. Past performance is not indicative and not a guarantee of future results. Indices shown are unmanaged and not available for direct investment. Performance data does not factor in transaction costs or taxes.
Cracks in the Bell Curve: Global Equities

Source: Morningstar EnCorr, MSCI

World ($)
Covariation of Returns: Linear or Nonlinear?

Source: Morningstar® EnCorr ® Stocks, Bonds, Bills, and Inflation module, MSCI
Tame vs. Wild Randomness

- Tame Randomness
  - Image an auditorium full of randomly selected people.
  - What do you estimate the average weight to be?
  - Now image the largest person that you can think of enters.
  - How much does your estimate change?
Tame vs. Wild Randomness

- Wild Randomness
  - Image an auditorium full of randomly selected people.
  - What do you estimate the average wealth to be?
  - Now image the wealthiest person that you can think of enters.
  - How much does your estimate change?
### Comparison of Asset Class Assumptions Models

<table>
<thead>
<tr>
<th></th>
<th>Lognormal</th>
<th>Johnson</th>
<th>Log-TLF</th>
<th>Bootstrapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parametric</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Flexible shape</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Scalable</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Randomness</td>
<td>Tame</td>
<td>Tame</td>
<td>Wild</td>
<td>NA</td>
</tr>
<tr>
<td>Covariation</td>
<td>Log-linear</td>
<td>Gaussian</td>
<td>Conditional Log-Linear</td>
<td>Non-linear</td>
</tr>
</tbody>
</table>
The Log-Stable Distribution


Source: Paul D. Kaplan, “Déjà Vu All Over Again,” in Morningstar Advisor, February/March 2009
Performance data shown represents past performance. Past performance is not indicative and not a guarantee of future results. Indices shown are unmanaged and not available for direct investment. Performance data does not factor in transaction costs or taxes.
The Left Tail of the Log-Stable Distribution


Source: Paul D. Kaplan, “Déjà Vu All Over Again,” in Morningstar Advisor, February/March 2009
Performance data shown represents past performance. Past performance is not indicative and not a guarantee of future results. Indices shown are unmanaged and not available for direct investment. Performance data does not factor in transaction costs or taxes.
Comparing Distributions: Global Equities

Source: Morningstar EnCorr, MSCI
Modelling Covariation
95% Confidence regions under alternative models

Measuring Long-Term Reward
Investment Horizon: One Period or Longer?

Payout from $1 investment for 3 choices
Meet the Choices

A
Average Payout $1.50
Geometric mean = $1.41

B
Average Payout $1.67
Geometric mean = $0.00

C
Average Payout $1.75
Geometric mean = $1.22

Meet the Choices

A

Average Payout
$1.50

B

Average Payout
$1.67

C

Average Payout
$1.75

Geometric mean = $1.41

Geometric mean = $0.00

Geometric mean = $1.22
Meet the Choices
Kelly Criterion: Rank Alternatives by Geometric Mean

A
Average Payout $1.50

B
Average Payout $1.67

C
Average Payout $1.75

Geometric mean = $1.41
Geometric mean = $0.00
Geometric mean = $1.22
Why the Kelly Criterion Works
Cumulative Probability Distribution after Reinvesting 12 Times
Measuring Risk with VaR & CVaR

- Value at Risk (VaR) describes the tail in terms of how much capital can be lost over a given period of time.
- A 5% VaR answers a question of the form:
  - Having invested 10,000 euros, there is a 5% chance of losing X euros in T months. What is X?
- Conditional Value at Risk (CVaR) is the expected loss of capital should VaR be breached.
- CVaR > VaR
- VaR & CVaR depend on the investment horizon.
Value-at-Risk (VaR)

VaR identifies the return at a specific point (e.g. 1\textsuperscript{st} or 5\textsuperscript{th} percentile)

- **Worst 1\textsuperscript{st} Percentile**: 99% of all returns are better, 1% of all returns are worse
- **Worst 5\textsuperscript{th} Percentile**: 95% of all returns are better, 5% of all returns are worse
Conditional Value-at-Risk (CVaR)

CVaR identifies the probability weighted return of the entire tail

Worst 5\textsuperscript{th} Percentile
95% of all returns are better
5% of all returns are worse
**CVaR vs. VaR**

Notice that different return distributions can have the same VaRs, but different CVaRs.

**Worst 5th Percentile**
95% of all returns are better
5% of all returns are worse
Markowitz 2.0
The Spirit of the Markowitz 2.0 Framework

- Go beyond traditional definition of good (expected return) and bad (variance)
- Use any definition of good
- Use any definition of bad
- Use any distributional assumptions (parametric or non-parametric)
Building A Better Optimizer

<table>
<thead>
<tr>
<th>Issue</th>
<th>Markowitz 1.0</th>
<th>Markowitz 2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return Distributions</td>
<td>Mean-Variance Framework (No fat tails)</td>
<td>Scenarios + Smoothing (Fat tails possible)</td>
</tr>
<tr>
<td>Return Covariation</td>
<td>Correlation Matrix Linear</td>
<td>Scenarios + Smoothing Nonlinear (e.g. options)</td>
</tr>
<tr>
<td>Investment Horizon</td>
<td>Single Period Arithmetic Mean</td>
<td>Can use Multiperiod Kelly Criterion</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Can use Geometric Mean</td>
</tr>
<tr>
<td>Risk Measure</td>
<td>Standard Deviation</td>
<td>Can use Conditional Value at Risk and other risk measures</td>
</tr>
</tbody>
</table>
## Markowitz 1.0 Inputs: Summary Statistics

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.00%</td>
<td>10.00%</td>
<td>1.00</td>
<td>0.34</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>B</td>
<td>10.00%</td>
<td>20.00%</td>
<td>0.34</td>
<td>1.00</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>C</td>
<td>15.00%</td>
<td>30.00%</td>
<td>0.32</td>
<td>0.82</td>
<td>1.00</td>
<td>0.71</td>
</tr>
<tr>
<td>D</td>
<td>13.00%</td>
<td>30.00%</td>
<td>0.32</td>
<td>0.82</td>
<td>0.71</td>
<td>1.00</td>
</tr>
</tbody>
</table>
## Scenario Approach to Modeling Return Distributions

<table>
<thead>
<tr>
<th>Scenario #</th>
<th>Economic Conditions</th>
<th>Stock Market Return</th>
<th>Bond Market Return</th>
<th>Real Estate Return</th>
<th>60/30/10 Mix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Low Inflation, Low Growth</td>
<td>5%</td>
<td>4%</td>
<td>4%</td>
<td>4.6%</td>
</tr>
<tr>
<td>2</td>
<td>Low Inflation, High Growth</td>
<td>15%</td>
<td>6%</td>
<td>11%</td>
<td>11.9%</td>
</tr>
<tr>
<td>3</td>
<td>High Inflation, Low Growth</td>
<td>-12%</td>
<td>-8%</td>
<td>-2%</td>
<td>-9.8%</td>
</tr>
<tr>
<td>4</td>
<td>High Inflation, High Growth</td>
<td>6%</td>
<td>0%</td>
<td>3%</td>
<td>3.9%</td>
</tr>
</tbody>
</table>

In practice, 1,000 or more scenarios typical so that fat tails and nonlinear covariations adequately modeled
## Scenarios Can be Added to Existing Models

- Tower Watson’s Extreme Risk Ranking at 30 June 2011

<table>
<thead>
<tr>
<th>1. Depression</th>
<th>2. Sovereign default</th>
<th>3. Hyperinflation</th>
</tr>
</thead>
</table>

Markowitz 2.0 Inputs: Scenarios
A Markowitz 2.0 Efficient Frontier
Read More About These and Other Ideas in My Book

“The breadth and depth of the articles in this book suggest that Paul Kaplan has been thinking about markets for about as long as markets have existed.”

From the foreword